

Pion-nucleus reactions in a microscopic transport model*

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Abstract

We analyse pion–nucleus reactions in a microscopic transport model of the BUU type, which propagates nucleons, pions, deltas and $N(1440)$ -resonances explicitly in space and time. In particular we examine pion absorption and inelastic scattering cross sections for pion kinetic energies $T_\pi = 85 - 315 MeV$ and various target masses. In general, the mass–dependence of the experimental data is well described for energies up to the Δ -resonance ($\approx 160 MeV$) while the absorption cross sections are somewhat overestimated for the higher energies. In addition we study the possible dynamical effects of delta– and pion–potentials in the medium on various observables as well as alternative models for the in-medium Δ -width.

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1 INTRODUCTION

Pion production in nucleus-nucleus reactions has been proposed in the early 80's as an observable to test the nuclear equation of state [1, 2, 3]. Since then, and also for more general questions about the properties of hadronic matter at high density, a lot of experimental and theoretical studies on pion production in heavy-ion collisions have been performed [2, 3, 4, 5, 6]. Nowadays it is widely accepted that pion production is not the most suitable process for studying the nuclear equation of state (EOS), especially in inclusive experiments, since the pions, which are created in the hot and dense reaction zone, are absorbed and reemitted several times before they leave the system such that the final pions emerge from densities zones $\rho \approx 0.5\rho_0$ [7]. Nevertheless, pion production is still an important field of present nuclear research due to the pion's important role in the overall reaction dynamics and the equilibration of the system.

The most elaborate theoretical approaches for the description of pion production are microscopical kinetic models which include the propagation of pions and nucleon-resonances as well as their mutual interactions [8, 9]. Basically there are three different microscopical realizations: the Intra Nuclear Cascade (INC)[10, 11, 12, 13], Quantum Molecular Dynamics (QMD) [14, 15], and the Boltzman-Uehling-Uhlenbeck model (BUU)[16, 17, 18, 19, 20]. In the INC model the nucleus-nucleus collision is simulated as the sum of all individual nucleon-nucleon collisions without taking into account self consistent mean-field potentials and Pauli blocking for the collisions. The QMD follows the same scheme as the INC, but takes into account the Pauli blocking in the collisions and a nucleus potential which is calculated as the sum of all two-body potentials. In this paper we will use the BUU-model [7] which is quite successful in describing the experimental data on pion production in proton-nucleus as well as nucleus-nucleus collisions. Since the pion spectra and pion yields turn out to be quite insensitive to details of the treatment of the $\pi N\Delta$ -dynamics in the BUU codes for heavy-ion reactions, we study in this paper explicitly pion-nucleus

reactions. We note that similar studies have already been performed with the INC model several years ago [21, 22, 23, 13].

From the pion physics point of view there has been a vivid interest in describing pion–nucleus reactions, because pion–nucleus reactions are expected to provide information on the properties of the strong interaction and excited states of the nucleon in the medium. Two major aspects are important for the understanding of pion–nucleus reactions: first, nuclear structure effects are important when looking for exclusive processes, i.e. detecting also the final state of the nucleus, and, second, the reaction mechanism itself which determines also inclusive reactions. Especially the question of the absorption mechanism is widely discussed [24, 25, 26, 27] in connection with multi–nucleon contributions and the properties of the delta resonance in the nuclear medium.

The most successful model up to date to describe inclusive pion–nucleus data is the model from Salcedo et al. [27]. In a first step these authors calculate microscopically density–dependent probabilities for pion quasielastic scattering and pion absorption in the nucleus, also taking into account three–body processes. With these probabilities they perform a pion cascade calculation using nuclear density profiles without, however, propagating the nucleons (or Δ ’s) explicitly. We will compare our results with their calculations.

In this paper we have the twofold goal to test the validity of the pion and resonance dynamics of our BUU model and to investigate the reaction mechanism in pion–nucleus reactions. In this respect we compare our calculations with total and differential cross sections for inclusive measurements in the pion energy region $T_\pi = 85\text{MeV} - 315\text{MeV}$ for pion absorptive and inelastic processes. This energy regime is of particular interest since here the delta is predominantly excited in the nucleus. To address the above mentioned questions, we proceed in the following way: in section 2 we review the basic inputs of the BUU model, especially those which are relevant for pion–nucleus dynamics. In section 3 we discuss the results of our model in comparison with various experimental data, compare with previous calculations

and study the dependence of observables on the input of our model. In section 4 we give a summary.

2 THE EXTENDED BUU MODEL

2.1 Basic equations

The dynamical description of hadron-nucleus or nucleus-nucleus reactions is based on the equation of motion for the time evolution of the nucleon one-body phase-space distribution $f(\vec{r}, \vec{p}; t)$

$$\frac{\partial f(\vec{r}, \vec{p}_1; t)}{\partial t} + \frac{\vec{p}_1}{m} \nabla_{\vec{r}} f(\vec{r}, \vec{p}_1; t) - \nabla_{\vec{r}} U(\vec{r}) \nabla_{\vec{p}_1} f(\vec{r}, \vec{p}_1; t) = I[f(\vec{r}, \vec{p}_1; t)] \quad (1)$$

with

$$I[f(\vec{r}, \vec{p}_1; t)] = \frac{4}{(2\pi)^3} \int d^3 p_2 d^3 p_3 \int d\Omega_4 v_{12} \frac{d\sigma}{d\Omega} \delta^3(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4) \times (f_3 f_4 \bar{f}_1 \bar{f}_2 - f_1 f_2 \bar{f}_3 \bar{f}_4) \quad (2)$$

where $U(\vec{r})$ is the nucleon mean-field potential, $d\sigma/d\Omega$ is the differential nucleon-nucleon cross section, $\bar{f}_i = 1 - f(\vec{r}, \vec{p}_i; t)$ the Pauli blocking factors and v_{12} the relative velocity between nucleon 1 and 2. Eq. (1) is called the Boltzmann-Uehling-Uhlenbeck equation (also known as Vlasov-Uehling-Uhlenbeck, Boltzmann-Nordheim or Landau-Vlasov equation). Its static homogeneous solution is the Fermi-Dirac distribution function. Details of the derivation of eq. (1) can be found in ref. [28, 29]. Physically the lhs of eq. (1) represents a Vlasov equation for a gas of nucleons moving independently in the mean field U . The rhs of eq. (1) is the two-body collision integral $I[f]$ which describes changes of the phase-space distribution due to nucleon-nucleon collisions incorporating Pauli-blocking for the final nucleon states.

Above the pion threshold inelastic processes become more and more important in nucleon-nucleon collisions such that pions, Δ 's and even higher resonances have to be included in the model. Therefore, in [16, 17] we have extended the BUU model to

coupled transport equations for pions, etas, Δ -, $N(1440)$ - and $N(1535)$ -resonance distribution functions [16, 17] (see also [30]). Since for the present study only nucleons, deltas, $N(1440)$'s and pions play a role, we discard the explicit propagation of etas and higher resonances.

Schematically one can write down the coupled equations used in our model in the following way:

$$\begin{aligned}
Df_N &= I_{NN}^N + I_{N\Delta}^N + I_{NN^*}^N + I_{\Delta \rightarrow N\pi}^N + I_{N^* \rightarrow N\pi}^N \\
Df_\Delta &= I_{N\Delta}^\Delta + I_{\Delta \rightarrow N\pi}^\Delta \\
Df_\pi &= I_{\Delta \rightarrow N\pi}^\pi + I_{N^* \rightarrow N\pi}^\pi \\
Df_{N^*} &= I_{N^* \rightarrow N\pi}^{N^*} + I_{N^*N}^{N^*}
\end{aligned} \tag{3}$$

where Df always denotes the differential operator for the Vlasov equation and the collision term on the rhs of eq. (3) incorporates the following processes:

$$\begin{aligned}
I_{N\Delta}^N, I_{N\Delta}^\Delta & \quad N\Delta \leftrightarrow N\Delta \text{ and } \Delta N \leftrightarrow NN \\
I_{NN^*}^N, I_{NN^*}^{N^*} & \quad NN^* \leftrightarrow NN^* \text{ and } NN^* \leftrightarrow NN \\
I_{\Delta \rightarrow N\pi}^N, I_{\Delta \leftrightarrow N\pi}^\pi, I_{\Delta \rightarrow N\pi}^\Delta & \quad \Delta \leftrightarrow N\pi \\
I_{N^* \rightarrow N\pi}^N, I_{N^* \leftrightarrow N\pi}^\pi, I_{\Delta \rightarrow N\pi}^{N^*} & \quad \Delta \leftrightarrow N\pi,
\end{aligned} \tag{4}$$

denoting $N(1440)$ by N^* .

In the eqs. (3) all direct processes for pion production and pion absorption are neglected. Since it is known that about 20% of the pions in nucleon–nucleon collisions, especially at low energies, are produced in direct processes, we have studied the effect of the direct channel by taking into account also the collision terms

$$I_{NN \rightarrow NN\pi}^N \quad \text{and} \quad I_{\pi NN \rightarrow NN}^N \tag{5}$$

for the change of the nucleon distribution function and the terms

$$I_{NN \rightarrow NN\pi}^\pi \quad \text{and} \quad I_{\pi NN \rightarrow NN}^\pi \tag{6}$$

for the change of the pion distribution function due to collisions.

Since in pion–nucleus reactions in the energy regime considered single resonance excitations clearly dominate, we neglect resonance–resonance scattering which, however, is included for the nucleus–nucleus case [31].

2.2 The test–particle method

The standard way to solve the coupled nonlinear integro–differential equations is to discretize the distribution function, i. e. to substitute the continuous distribution functions by a finite number of test–particles, i. e.

$$f(\vec{r}, \vec{p}; t) = \frac{1}{N} \sum_i^{N \cdot A} \delta(\vec{r} - \vec{r}_i(t)) \delta(\vec{p} - \vec{p}_i(t)) \quad (7)$$

where N denotes the number of test-particles per nucleon and A denotes the total number of nucleons in the reaction. The ansatz (5) is a solution of the Vlasov equation if $\vec{r}_i(t)$ and $\vec{p}_i(t)$ follow the classical Hamilton-equations,

$$\begin{aligned} \frac{d\vec{p}_i}{dt} &= -\nabla_{\vec{r}} U, \\ \frac{d\vec{r}_i}{dt} &= \frac{\vec{p}_i}{m_i}. \end{aligned} \quad (8)$$

Thus the problem reduces to a classical time evolution of a system of a finite number of test-particles. By discretizing the time t we solve this simplified problem as follows: For given positions and momenta of the test-particles at time t their evolution at $t + \delta t$ reads:

$$\vec{p}(t + \delta t) = \vec{p}(t) - \delta t \nabla_{\vec{r}} U(\vec{r}, t), \quad (9)$$

$$\vec{r}(t + \delta t) = \vec{r}(t) + \delta t \left(\frac{\vec{p}(t)}{E} \right). \quad (10)$$

Instead of \vec{p}/m we use \vec{p}/E in the Hamilton-equation as appropriate for relativistic kinematics. We calculate the effect of collisions, resonance decays or absorptions between $t + \delta t/2$ and $t + 3 \delta t/2$ by assuming straight line trajectories during this time interval. For the simulation of inelastic reaction mechanisms we use the parallel ensemble algorithm (ref. [28]) in which collisions are only allowed between particles

in the same ensemble. We note that the isospin degrees of freedom are accounted for by separate distribution functions, but Coulomb-effects are not taken into account.

2.3 Mean-field potentials

The particles inside the nuclear medium propagate in different mean-field potentials as described in eqs. (1),(9),(10). The nucleon mean-field potential is evaluated selfconsistently on the basis of a density-dependent Skyrme potential as:

$$U_N(\rho) = 0.75t_0\rho + t_3\rho^\tau \quad (11)$$

with $t_0 = -1177.8\text{MeV}/\text{fm}^3$, $t_3 = 1845.5\text{MeV}/\text{fm}^5$ and $\tau = 5/3$, which leads to a binding energy of -16MeV per nucleon at $\rho_0 \approx 0.16\text{fm}^{-3}$ and an incompressibility $K = 308.4\text{MeV}$. These parameters give quite realistic, stable density distributions for finite nuclei in the surface region.

The delta and $N(1440)$ resonance are in general assumed to propagate within the same mean field as the nucleons if not explicitly noted otherwise. Deviations from this average potential will not only influence the propagation of the resonances R through the nucleus, but also change the collision probabilities; e.g. a difference between the nucleon and the resonance potentials will shift the energy in the $RN \rightarrow NN$ reaction and the peak position in the $\pi N \rightarrow R$ process. Thus to exploit the dynamical influence of an alternative delta potential we also incorporate different prescriptions for this mean field and discuss the implications in comparison to the experimental data.

In our simulations we use two approximations for the delta potential. The first model relates to elastic pion-scattering experiments where one finds a delta potential $V_\Delta \approx -30\text{MeV}$ at ρ_0 [32]. Thus we have employed a delta potential which is just the nucleon potential shifted to -30MeV at normal nuclear density,

$$U_\Delta(\rho) = 0.03 \frac{U_N(\rho)}{|U_N(\rho_0)|} (\text{GeV}) . \quad (12)$$

As a second model we used a delta potential as extracted by Eehalt et al. in [33] from the delta-hole model. A suitable parametrization of this delta potential is given by [33]:

$$U_{\Delta}(\rho) = -0.7\rho + 1.75\rho^{5/3}(GeV) \quad . \quad (13)$$

This potential is also $-30MeV$ at ρ_0 , but shows a different density dependence.

Furthermore, in line with the delta-hole model also the pion propagates in its own potential field in the medium. We thus consequently also use the respective pion potential as evaluated and parametrized in our former work [33], where we have investigated the effect of pion and delta potentials on asymptotic pion spectra and dilepton yields in heavy-ion collisions.

2.4 Simulation of the collision integrals

The test particles in our approach collide with each other as in conventional cascade simulations with collision rates as described by eq. (4). The reaction probabilities are calculated on the basis of free cross sections for the different processes and implemented as explained in detail in [16] taking into account the Pauli blocking of the nucleons in the final state with proper isospin factors. The time-step used in the calculations is $\Delta t = 0.5fm/c$; we did not find any significant change in the results when decreasing Δt further.

2.5 πN cross section

For the $\pi N \rightarrow \Delta$ cross section we employ a Breit-Wigner formula

$$\sigma_{\pi^+p \rightarrow \Delta^{++}} = \sigma_{\pi^+p}^{max} \left(\frac{q_r}{q} \right)^2 \frac{\Gamma(M)}{(M_{\Delta} - M)^2 + \frac{1}{4}\Gamma(M)^2} \quad (14)$$

with the momentum-dependent width [34]:

$$\Gamma(M) = \left(\frac{q}{q_r} \right)^3 \frac{M_{\Delta}}{M} \left(\frac{v(q)}{v(q_r)} \right)^2 \Gamma_r \quad , \quad (15)$$

where M is the actual delta mass, M_Δ is the peak delta rest mass ($1232MeV$), q is the pion momentum in the rest frame of the delta, q_r the value of q for a delta mass M_Δ and $\Gamma_r = 110MeV$. The function

$$v(q) = \frac{\beta^2}{\beta^2 + q^2} \quad (16)$$

with $\beta = 300MeV$ cuts the width at high momenta.

From the particle data table [35] we get

$$\begin{aligned} \sigma_{\pi^+p}^{max} &= 200mb \\ \sigma_{\pi^-p}^{max} &= 70mb \end{aligned} \quad (17)$$

and based on isospin invariance of the strong interaction we use

$$\begin{aligned} \sigma_{\pi^-n}^{max} &= 200mb \\ \sigma_{\pi^+n}^{max} &= 70mb \\ \sigma_{\pi^0n}^{max} = \sigma_{\pi^0p}^{max} &= 135mb \end{aligned} \quad (18)$$

in eq. (14) for the respective cross sections.

The production of the $N(1440)$ is only possible in the P_{11} channel. In order to reproduce the π^+n cross section around the $N(1440)$ resonance we use the following form of the cross section for the $\pi N \rightarrow N(1440)$ process

$$\sigma_{\pi^+n \rightarrow N^*} = \sigma_{\pi^-p \rightarrow N^*} = \sigma_{\pi N \rightarrow N^*}^{max} \frac{0.25\Gamma_{N^*}^2}{(M_{N^*} - M)^2 + \frac{1}{4}\Gamma_{N^*}^2} \quad (19)$$

with $\sigma_{\pi N \rightarrow N^*}^{max} = 30mb$ and $\Gamma_{N^*} = 200MeV$ which yields the cross section (dashed line) in fig. 1. The sum of Δ and $N(1440)$ cross sections (eq. (14) with eq. (18) and eq. (19)) then gives the correct value of the total cross section (cf. fig. 1 (dashed–dotted line)).

If a Δ or $N(1440)$ is created in a πN collision, its mass is determined by the invariant mass \sqrt{s} of the πN system; if it is created in a NN -collision, then its mass is randomly chosen weighted with the proper Breit–Wigner distribution (see eq. (29)). In our approach the resonances are treated as on-shell particles, however, with dynamically determined mass and lifetime (see below).

2.6 Decay of resonances

The decay of a resonance $R \rightarrow \pi N$ in free space is determined by its life time $\Gamma(M)^{-1}$. However, inside the nucleus the resonance decay may be forbidden by the Pauli-principle. In addition, a resonance can also decay by the process $RN \rightarrow NN$ due to the presence of other nucleons. We will discuss the latter channel in the next subsection and describe here the numerical implementation of the 'free' decay inside the nucleus.

Both resonances, Δ and $N(1440)$, decay with 99.5% and 65%, respectively, into a πN final state. Thus in every time-step we calculate the decay probability into a πN state in the rest frame of the resonance by assuming an exponential decay law

$$P_{decay} = e^{-\Gamma(M)\Delta t} \quad (20)$$

where $\Gamma(M)$ is the width of the resonance and Δt is the time-step in our calculation. By a Monte Carlo method we then decide, if the resonance actually decays in a given time-step. In case of a Δ we use eq. (15) for the width, for the $N(1440)$ a constant width $\Gamma = 200 MeV$. Before we let the resonance decay we check if the outgoing nucleon is Pauli blocked by counting the number of testparticles in the final phase space cell as described in [28].

The Δ decay, furthermore, is assumed to be isotropic in its rest frame and the pion and nucleon momenta are determined by energy and momentum conservation. Nevertheless, to test the influence of an anisotropic delta decay we also have performed calculations with a p-wave angular distribution for the differential πN cross section

$$\frac{d\sigma}{d\Omega} \sim (1 + 3 \cos^2 \Theta) \quad (21)$$

where Θ is the angle between the initial and final pion in the delta rest frame (c.f. section 3.3).

2.7 Direct pion emission and absorption

Whereas in the VerWest–Arndt parametrization for the $NN \rightarrow N\Delta \rightarrow NN\pi$ process ([36] c.f. section 2.8) all pions are assumed to be produced via a delta or a $N(1440)$ –resonance up to $\sqrt{s} \approx 2.5\text{GeV}$, the pion may also be created directly in a NN collision. We calculate the direct pion production cross section, which is the production via an off-shell nucleon, in lowest order perturbation theory using a One–Boson–Exchange model for the nucleon–nucleon interaction. The full theory, that takes into account intermediate deltas and nucleons, reproduces the experimental data rather well [37]. To extract the direct contribution to the production process we dropped all diagrams containing a delta in the intermediate state. (For a more detailed description of the calculations we refer the reader to ref. [37].)

For the different isospin channels we use the isospin decomposition of the $NN \rightarrow NN\pi$ process (according to ref. [36]):

$$\begin{aligned}
pp &\rightarrow pp\pi^0 & \sigma_{11} \\
pp &\rightarrow pn\pi^+ & \sigma_{11} + \sigma_{10} \\
np &\rightarrow np\pi^0 + & \frac{1}{2}(\sigma_{10} + \sigma_{01}) \\
np &\rightarrow nn\pi^+ + & \frac{1}{2}(\sigma_{11} + \sigma_{01}) \\
np &\rightarrow pp\pi^- + & \frac{1}{2}(\sigma_{10} + \sigma_{01})
\end{aligned} \tag{22}$$

where I, I' in $\sigma_{I,I'}$ denotes the initial and final isospin of the participating nucleon pair. By calculating the direct contribution to the $pp \rightarrow pn\pi^+$, $pp \rightarrow pp\pi^0$ and $np \rightarrow np\pi^0$ processes we determine all isospin channels using decomposition eq. (22). Our result for the direct $pp \rightarrow np\pi^+$ process is shown in fig. 2 (solid line). It has the following functional dependence on \sqrt{s} (in GeV):

$$\sigma_{pp \rightarrow np\pi^+}^s(\sqrt{s}) = 26.2 - 40.6712\sqrt{s} + 13.55566s \quad . \tag{23}$$

For the other isospin channels we used the same energy dependence with the following ratios (calculated at $\sqrt{s} = 2.155\text{GeV}$):

$$\sigma_{pp \rightarrow np\pi^+} : \sigma_{np \rightarrow np\pi^0} : \sigma_{pp \rightarrow pp\pi^0} = 6.98 : 2.269 : 1 \quad . \tag{24}$$

Taking into account the direct pion production process, we simultaneously have to consider the inverse process $\pi NN \rightarrow NN$. We avoid the problem of calculating a cross section with a three-particle initial state by using the quasi 'deuteron' assumption; the pions are assumed to be absorbed on nucleon pairs as in the conventional picture for pion absorption [25]. The $\pi(2N) \rightarrow NN$ reaction we then can calculate from the direct production cross sections by using detailed balance:

$$\sigma_{\pi NN \rightarrow NN} = \frac{4 p_f^2}{3 p_i^2} \sigma_{NN \rightarrow NN \pi}^s \quad (25)$$

with

$$p_i^2 = (s - (m_\pi - 2m_n)^2)(s - (m_\pi + 2m_n)^2)/(4s) \quad (26)$$

$$p_f^2 = \frac{s}{4} - m_n^2 \quad , \quad (27)$$

which denote the initial $(2N)$ pair and final nucleon momenta in the center of mass frame. In fig. 2 we show the $\pi^+(pn) \rightarrow pp$ cross section used in the calculation by the dashed line. The isospin dependence discussed above leads to the following absorption ratios:

$$\begin{aligned} \frac{\sigma_{\pi^+(nn) \rightarrow np}}{\sigma_{\pi^+(np) \rightarrow pp}} &= 0.083 \\ \frac{\sigma_{\pi^0(np) \rightarrow np}}{\sigma_{\pi^+(np) \rightarrow pp}} &= 0.44 \\ \frac{\sigma_{\pi^0(nn) \rightarrow nn}}{\sigma_{\pi^+(np) \rightarrow pp}} &= 0.14 \\ \frac{\sigma_{\pi^0(pp) \rightarrow pp}}{\sigma_{\pi^+(np) \rightarrow pp}} &= 0.14 \\ \frac{\sigma_{\pi^-(pp) \rightarrow np}}{\sigma_{\pi^+(np) \rightarrow pp}} &= 0.083 \\ \frac{\sigma_{\pi^-(np) \rightarrow nn}}{\sigma_{\pi^+(np) \rightarrow pp}} &= 1 \quad . \end{aligned} \quad (28)$$

2.8 $N\Delta \leftrightarrow NN$ and $NN(1440) \leftrightarrow NN$ reactions

For the $NN \rightarrow N\Delta$ and $NN \rightarrow NN(1440)$ processes we use the VerWest–Arndt parametrization of the cross section as described in detail in ref. [16]. The mass of

the resonance R , which is populated in our simulation in a NN -collison, is chosen according to the probability function

$$F(M^2) = \frac{1}{\pi} \frac{M_\Delta \Gamma(M)}{(M^2 - M_\Delta^2)^2 + M_\Delta^2 \Gamma(M)^2} \quad (29)$$

with $\Gamma(M)$ for deltas given in eq. (15) and $\Gamma(M) = 200 MeV$ for $N(1440)$. In a pion-nucleus reaction these resonance production processes are negligible, but the inverse processes $N\Delta \rightarrow NN$ and $N(1440)N \rightarrow NN$ are very important in the context of pion absorption. In the literature the latter reactions are determined from the measured cross sections for the inverse channels by detailed balance using slightly different assumptions [7, 17, 28, 38], that have to be discussed in more detail. The ambiguity arises because the measured cross sections given in ref. [36] are averaged over the Δ -mass distribution.

Application of detailed balance to a hypothetical delta with fixed mass leads to [28]

$$\sigma_{\Delta^{++}n \rightarrow pp} = \frac{1}{4} \frac{p_f^2}{p_i^2} \sigma_{pp \rightarrow n\Delta^{++}} \quad (30)$$

for the $\Delta^{++}n \rightarrow pp$ cross section. In eq. (30) p_f, p_i are the final and initial momenta of the particles in the center of mass frame whereas the factor $1/4$ is due to spin averaging and a symmetry factor for identical particles in the final state.

Taking into account, however, that the delta has no fixed mass in our simulation, we get[17]:

$$\sigma_{n\Delta^{++} \rightarrow pp}^{tot} = \frac{2\pi}{4} \frac{p_N^2}{p_\Delta^2} \frac{1}{F(M^2)} \int_{-1}^1 \frac{d\sigma_{pp \rightarrow n\Delta^{++}}}{d(\cos \Theta) dM^2} d(\cos \Theta) \quad (31)$$

Since the VerWest-Arndt parametrization only gives the Δ -mass averaged total pion production cross section it cannot be used to calculate the inverse cross section in eq. (31). Following our former work [17] we therefore use

$$\sigma_{n\Delta^{++} \rightarrow pp} = \frac{4\pi}{4} \frac{p_N^2}{p_\Delta^2} \sigma_{pp \rightarrow n\Delta^{++}} \frac{1}{\int_{(m_N+m_\pi)^2}^{(\sqrt{s}-m_N)^2} f(M^2) dM^2} \quad (32)$$

with the $NN \rightarrow N\Delta$ cross sections from VerWest-Arndt.

To clarify the situation concerning the different expressions in the literature for the $N\Delta \rightarrow NN$ reaction we inserted in eq. (31) the result for the mass-dependent cross section $d\sigma_{pp \rightarrow n\Delta^{++}}/(d\Omega dM^2)$ from a calculation of Schäfer et al. [37]. The latter differential cross section for the $pp \rightarrow n\Delta^{++}$ process is calculated on the basis of a OBE model using pion- and ρ -exchange which reproduces the experimental data for $d\sigma/dM$ (cf. fig. 3) remarkable well.

To investigate the validity of our model assumption in [17] we compare the results of eqs. (30),(32) and eq. (31) calculated with the OBE model for the $\Delta^{++}n \rightarrow pp$ cross section for different delta masses $M = 1180, 1232$ and $1280 MeV$ in fig. 4. Our cross section (solid line) eq. (32) is in good agreement with the OBE result (dotted line) for larger invariant energy \sqrt{s} , but overestimates the absorption cross section close to threshold. In fig. 4 also the cross section as proposed by Danielewicz and Bertsch in ref. [38] is shown by the dashed-dotted line. It fits better at small \sqrt{s} but overestimates the absorption significantly at higher energy \sqrt{s} . The naive detailed balance (30) (dashed line) underestimates the cross section at almost all energies. For the dynamical implications of the different models in pion-nucleus reactions we refer the reader to sections 3.1 and 3.2.

Finally, using isospin factors and taking into account symmetry factors for identical particles in the final state, we obtain the total cross sections of the other isospin channels as

$$\begin{aligned}\sigma_{n\Delta^+ \rightarrow pn}^{tot} &= \sigma_{p\Delta^0 \rightarrow pn}^{tot} = \frac{2}{3}\sigma_{n\Delta^{++} \rightarrow pp}^{tot} \\ \sigma_{p\Delta^+ \rightarrow pp}^{tot} &= \sigma_{n\Delta^0 \rightarrow nn}^{tot} = \frac{1}{3}\sigma_{n\Delta^{++} \rightarrow pp}^{tot} \\ \sigma_{p\Delta^- \rightarrow nn}^{tot} &= \sigma_{n\Delta^{++} \rightarrow pp}^{tot} \quad .\end{aligned}\tag{33}$$

In case of the $NN(1440) \rightarrow NN$ process we use eq. (32) with a constant width $\Gamma_{N^*} = 200 MeV$ and the $NN \rightarrow NN(1440)$ cross section from the VerWest-Arndt parametrization. Additionally, we also check for Pauli blocking of the outgoing nucleons; however, due to the large momenta transferred to both nucleons the blocking effect is found to be of minor importance in pion-nucleus reactions.

2.9 NN , $N\Delta$ and $N(1440)N$ elastic scattering

For these processes we use the conventional Cugnon parameterization [11, 28]

$$\sigma(mb) = \frac{35}{1 + 100\sqrt{s'}(GeV)} \quad (34)$$

with $\sqrt{s}' = \sqrt{s} - 1.8993$ for a NN collision and $\sqrt{s}' = \sqrt{s} - 0.938 - M_R$ for nucleon-resonance elastic scattering. The anisotropic angular distribution used is adopted from [16].

2.10 Inclusive cross sections

To calculate inclusive cross sections for a pion-nucleus reaction we perform an explicit impact parameter integration,

$$\sigma = \int 2\pi b db \ N_{reac}(b) \quad , \quad (35)$$

where $N_{reac}(b)$ denote the impact-parameter dependent particle multiplicities as obtained from simulations with fixed b for the pions.

This technique is identical to the one employed for proton-nucleus or nucleus-nucleus reactions so that we describe all reactions on the same footing. In ref. [16, 33] we have already shown that the experimental spectra on pion production in heavy-ion collisions can be reproduced very well, except for some small differences at small pion transverse momenta. If this also holds for pion-nucleus collisions will be studied in the next section.

3 RESULTS FOR PION-NUCLEUS REACTIONS

3.1 Total absorption cross sections

As noted in the introduction the mechanism of pion absorption in nuclei is not well understood. Energy and momentum conservation rule out absorption of a pion

on a free single nucleon. However, due to the large momentum mismatch in the reaction the single-nucleon absorption is very improbable in pion-nucleus reactions, too. The most important mechanism is thus the two-body absorption, where the pion is absorbed by a pair of nucleons. The quantitative contribution of higher order processes and especially that of three-body processes is still a matter of debate. For heavier nuclei the three-body process is controversially discussed to contribute between 10% and 50% of the total absorption cross section [26].

In the transport simulations the pion absorption mainly proceeds via a two-step mechanism: a pion first is captured in a Δ state and this Δ can be absorbed in a second step via the channel $\Delta N \rightarrow NN$. This two-step process clearly simulates a two-body absorption mechanism. However, by allowing the Δ to decay to πN and the outgoing pion to be captured again in another Δ (subsequent two-body reaction chain), we incorporate also higher order absorption mechanisms with on-shell intermediate pions.

In fig. 5 we show the calculated mass dependence of the pion absorption cross section at the pion energy $T_\pi = 165 MeV$ in comparison to the experimental data. To illustrate the influence of the $\Delta N \rightarrow NN$ reaction we have performed calculations with the different detailed balance prescriptions discussed in section 2.8. We find that our previously used formula eq. (32) [17](solid line), the OBE results eq. (31) (dotted line) and the prescription from ref. [38](dashed-dotted line) lead to the same accuracy in reproducing the experimental data, whereas the naive detailed balance (eq. (30)) misses about 50% of the cross section. This reflects the fact, that the detailed balance prescription of eq. (30) underestimates the elementary absorption cross section over the whole energy regime (see fig. 4) and explains why the cascade calculation of [13] underestimated the absorption cross section by a factor of 2.

Turning now to the energy dependence of the absorption cross section (fig. 6), we find that we can describe the cross section at low energies up to the resonance region very well for all target masses. For higher energies ($T_\pi = 315 MeV$), however, our calculation (solid line) overestimates the data. This disagreement becomes larger for

heavier nuclei. It is worthwhile to note that even in this T_π regime the A -dependence of the absorption cross section is well described. The energy dependence of the pion absorption is not altered when using the OBE model (dotted line) or the prescription from ref. [38](dashed-dotted line) (see section 2.8) for the $\Delta N \rightarrow NN$ reaction.

Compared to other calculations for the total absorption cross section as a function of target mass and kinetic energy of the pion [23, 27] our results are of comparable quality. The INC calculations have problems in reproducing the mass dependence of the total absorption cross section [23] and also the absolute value in the resonance region, whereas the calculations of Salcedo et al. [27] also overestimate pion absorption at the higher pion energies for heavier nuclei.

To simulate the effect of a direct three-body absorption process in our approach, we used a density dependent modification for the $\Delta N \rightarrow NN$ cross section of the form:

$$\sigma'_{\Delta N \rightarrow NN} = (1 + 3\frac{\rho}{\rho_0})\sigma_{\Delta N \rightarrow NN} \quad , \quad (36)$$

thus incorporating the density dependence of three-body pion absorption in an ad-hoc way. The result for the energy-dependent absorption cross section on ^{12}C and ^{209}Bi with the density-dependent cross section (36) is shown in fig. 7 (dashed line). We find only a minor three-body effect for the ^{209}Bi case. This is due to the large diameter of the Bi -nucleus; most deltas inside the Bi -nucleus scatter several times with nucleons and are absorbed. An increase of the $\Delta N \rightarrow NN$ cross section as introduced in eq. (36) just leads to an earlier absorption and not to an increase of the total pion absorption cross section.

For the ^{12}C nucleus we find a bigger effect of the three-body absorption process as simulated by prescription (36); fig. 7 (dashed line). In this lighter nucleus deltas do not scatter as often as in the Bi -case and can leave the nucleus without being absorbed. Since the escape probability of the Δ 's increases with their velocity, modifications of eq. (36) for the $\Delta N \rightarrow NN$ process affect the total pion absorption cross section dominantly at higher energies, thus overestimating the total pion absorption cross section even more.

Salcedo et al. [27] report a larger contribution of the three-body absorption processes of about 30%, but these calculations would lead to similar results as ours, if they omitted their three-body absorption probabilities. Similar to our case without three-body absorption probabilities the pions in their calculation would be absorbed further inside the nucleus via a two-body process. We thus conclude that the total inclusive pion absorption process in heavy nuclei is not sensitive to true many-body effects. It seems that subsequent two body steps, taking into account off-shell deltas, can simulate the many-body effects.

We additionally note, that we have also studied the influence of the $N(1440)$ for the absorption process and have found only a 10% difference for $T_\pi \approx 300 MeV$ when performing the calculations with and without the $N(1440)$ resonance. For smaller energies the contribution of the $N(1440)$ is negligible.

As mentioned in section 2.7 we studied also the effect of direct (NN) pion production and absorption. We find a 10% enhancement for the absorption cross section at all pion energies in the ^{12}C case, whereas in the ^{209}Bi case the pions are absorbed earlier in the reaction due to the incorporation of the direct process, but the total absorption cross section is not changed (see discussion on many-body absorption above).

Summarizing this section we may state that we reproduce the experimental data on pion absorption quite well, for light nuclei at all energies and for heavy nuclei for energies up to the Δ -resonance region; for heavy nuclei above the Δ -resonance we obtain a too large cross section. The origin of this deficiency may be in the cascade description itself. We find that for central (π, A) collisions the absorption cross section even has a minimum at $T_\pi = 160 MeV$ because at this resonance energy the Δ -excitation cross section is very large and the delta can thus be formed already in the nuclear surface, where the probability for the decay pion to escape the nucleus again (inelastic scattering) is very large. The calculated energy dependence of the absorption cross section, which shows a different behaviour, is thus largely determined by the impact parameter dependence.

3.2 Total inelastic cross section

Inelastic pion-nucleus scattering occurs in competition with the absorption process. Thus to understand the pion-nucleus reaction both processes must be described within the same approach.

As shown in fig. 8 our calculations reproduce the mass dependence of the total inelastic cross section for pion energies ($T_\pi = 165\text{MeV}$) (solid line). This also holds when employing the OBE model for the $\Delta N \rightarrow NN$ channel (dotted line) or the prescription by Danielewicz [38] (dashed-dotted line).

When looking at the energy dependence of the inelastic cross sections, which is shown in fig. 9 for ^{12}C and ^{209}Bi , we see that for the ^{209}Bi case we underestimate the cross section by about a factor 2 at all pion energies; for ^{12}C this only happens for higher energies above resonance.

A detailed phase-space analysis offers the following kinematic reason for these deficiencies: Generally most of the pions that are scattered inelastically emerge from the first Δ generation which is dominantly created in the surface of the nucleus; approximately 50% of the pions from the first Δ generation leave the system again and are not absorbed. When increasing the pion kinetic energy above the resonance region the cross section for delta formation becomes smaller and consequently the first delta generation is created further inside the nucleus, such that pions from its decay cannot leave the nucleus as easily as in the resonance region. Furthermore, these secondary pions are shifted down in energy (see section 3.5), which leads to an effective increase of the reabsorption cross section. As a consequence the secondary pions in the interior of the nucleus are almost completely absorbed, which leads to a reduced inelastic pion cross section and a correspondingly enhanced absorption cross section for higher incident pion energies.

At low energies, on the other hand, the first generation of deltas has a larger life time due to their low mass (see section 3.5). This does not change the picture just described in heavy nuclei, but in light nuclei some of these deltas can thus decay

outside the nucleus and the secondary pions can escape again.

3.3 Differential inelastic cross section

Since in the resonance region the inelastic cross sections are well reproduced, we proceed with calculating also differential cross sections to get further information on the initial delta distributions. In fig. 10 the results of our simulations for $\pi^+ + {}^{12}\text{C}$ and $\pi^+ + {}^{209}\text{Bi}$ at $T_\pi = 160\text{MeV}$ are shown for the differential inelastic cross section (solid line). Except for the forward angles the data for ${}^{209}\text{Bi}$ are rather well reproduced both with an isotropic and an anisotropic delta decay (see discussion in section 2.6). For the ${}^{12}\text{C}$ target, however, the anisotropic delta decay gives clearly a better description of the data at backward angles (dashed line in fig. 10). The deficiency in forward direction is due to the fact that the forward scattered pions will be captured in a delta again so that it becomes rather improbable to leave the nucleus in forward direction. Furthermore, the forward amplitude is also expected to be sensitive to coherent quantum scattering [27]; this effect is completely missing in our incoherent transport approach.

Using the more realistic anisotropic delta decay instead of the isotropic one we find that this effects the total cross section for absorption and inelastic scattering by at most 5% such that the basic conclusions of the previous sections remain unchanged. Again our results are comparable with those of Salcedo et al. (ref. [27]) and are quantitatively better than the results from INC calculations [23].

3.4 (π^+, Np) reactions

A further test for the absorption mechanism in our model is the comparison to more exclusive experiments where also the continuum protons are detected in a pion absorption event. In a recent study Ransome et al. [39] present total cross sections for pion absorption on nuclei followed by 2p,3p emission for different energies and masses. We have calculated the $(\pi^+, 2p)$ and $(\pi^+, 3p)$ cross sections for $T_\pi = 150\text{MeV}$, where

element	$(\pi^+, 2p\&3p)$	$(\pi^+, 2p\&3p)$	$(\pi^+, 3p)$	$(\pi^+, 3p)$
	BUU	exp.	BUU	exp.
^{12}C	114	110	15	14
^{58}Ni	272	300	30	33
^{118}Sn	333	320	46	31
^{208}Pb	399	400	48	24

Table 1: Cross sections for (π^+, Np) reactions in mb for $T_\pi = 150MeV$ from ref. [39]. We compare with the sum of $2p$ and $3p$ events, because these are the experimental data given under the heading total $2p$.

our calculation reproduces the total absorption cross section. For the continuum protons we applied the same cut in energy $E_p > 23MeV$ as in the experiment. To calculate the cross section we proceeded as described in section 2.10. For different impact parameters we counted the absorption events followed by two, three proton emission, respectively.

Our calculated results are compared in table 1 with the data; they are in good agreement, especially for the lighter nuclei. For the heavier nuclei we get too many $(\pi^+, 3p)$ events, but for this channel the errors in the experimental data are bigger. In accordance with the experimental data we find that the contribution of (π^+, Np) processes with $N > 3$ to the absorption is negligible.

Thus we conclude that the average event characteristics are well described within our transport approach.

3.5 Pion and delta dynamics

In the calculations described so far the deltas have been propagated in the same potential as the nucleons. Recently [13, 33, 40], there has been some discussion of the properties of the delta in the nuclear medium. In order to explore the physical consequences of any in-medium changes we have performed calculations changing

artificially the peak mass and width of the resonance. To be more precise, in the width $\Gamma_{\Delta}(M)$ we have changed the parameter Γ_r in eq. (15), which consequently influences all other cross sections that depend on $\Gamma(M)$. In varying systematically the peak mass of the delta and its width at maximum we find that both changes do not improve the energy-dependence of the absorption cross section discussed earlier in sections 3.1 and 3.2. The best reproduction of the absorption data is achieved for the free width and the free delta peak mass (solid line in fig. 11). Thus our model analysis of pion absorption cross sections indicates no need for any in-medium changes of the Δ -resonance.

In a recent publication Sneppen and Gaarde [40] have argued that the analysis of charge exchange reactions in terms of a cascade model requires an in-medium delta mass of $M_{\Delta} = 1202 MeV$ and a width of $\Gamma(M_{\Delta}) = 200 MeV$. In a calculation with these parameters we find for the ^{12}C target (fig. 12, (dashed line)) that the agreement with the experimental data at higher energies is better compared to the original calculation (solid line), but becomes worse for low T_{π} . In addition, the inelastic scattering cross section at higher energies is not well described. For the ^{209}Bi case (not explicitly displayed) there is not much change.

Another in-medium effect for the deltas is the delta potential as already pointed out in section 2.3. In order to explore the sensitivity of the pion-nucleus data to pion and delta selfenergies we have studied the influence of pion and delta potentials given in section 2.3 in our simulations. Whereas the effect of a pion potential changes the result only on a 5% level (not explicitly displayed), the pion absorption shows a larger sensitivity to a Δ -potential as shown in fig. 13. We find that the effective potential from the delta-hole model eq. 13 (dotted line) and the simple shifted potential eq. (12) (dashed line) lead to similar results. The calculated changes in the absorption cross section can be directly traced back to the energy shift in the πN reaction.

Furthermore, to separate kinematical effects in the delta and pion dynamics in the pion-nucleus reactions we have calculated the differential Δ mass distribution for different Δ generations. The results in fig. 14 suggest the following picture: At

higher energies also high mass deltas are excited, as naively expected. These deltas in turn have a short life time (large $\Gamma(M)$) and decay fast. The pions from their decay are reabsorbed again and regenerate the Δ 's although now at a smaller mass due to recoil effects. Thereby the Δ 's cascade down in mass and loose about 70MeV in mass per generation.

4 SUMMARY

We have analysed pion–nucleus reactions within the extended BUU model that had originally been developed for the description of nucleus-nucleus collisions. Due to the pionic entrance channel we expected to be especially sensitive to the pion and delta dynamics employed and thus to be able to put a stringent constraint on these ingredients for the description of heavy–ion collisions.

Apart from heavy–ion collisions we can reproduce the inclusive data for pion-nucleus reactions for a wide range of pion kinetic energies and target masses. Within the transport approach we have simulated pion absorption by sequential binary reactions between pions, nucleons and delta's. The measured pion absorption cross sections are very well reproduced for pion energies up to the resonance energy; for higher energies we overestimate the absorption. The mass dependence, on the other side, is very well reproduced. The dominant part of the many–body absorption effects seems to be simulated by the dynamics incorporated in our transport approach. The deficiencies of the model could essentially be traced back to the first generation of deltas.

Apart from pion absorption we also reproduce the inelastic cross sections for light nuclei up to the resonance region quite well. For heavy nuclei we seem to underestimate the inelastic cross section at all energies.

We obtain a good agreement for differential inelastic scattering and the exclusive (π^+, Np) reactions in the delta resonance region which demonstrates that the overall event pattern is described quite well in this region.

In summary we find that the BUU transport approach developed for nucleus-nucleus collisions provides a rather reliable description of pion-nucleus reactions, too. The results obtained for the different channels in the pion-nucleus reactions are in their quality comparable to those of other recent model calculations for these reactions.

References

- [1] W. Greiner H. Stöcker and W. Scheid, *Z. Phys.* **A286** (1978) 121.
- [2] R. Stock et al., *Phys. Rev. Lett.* **49** (1982) 1236.
- [3] R. Stock, *Phys. Rev.*, **135** (1986) 256.
- [4] J.W. Harris et al., *Phys. Lett.* **153B** (1985) 377.
- [5] J.W. Harris et al., *Phys. Rev. Lett.* **58** (1987) 463.
- [6] B. Waldhauser et al., *Z. Phys.* **A328** (1987) 19.
- [7] Gy. Wolf, W. Cassing, and U. Mosel, *Nucl. Phys.* **A552** (1993) 549.
- [8] U. Mosel, *Ann. Rev. Nucl. Part. Sci.* **41** (1991) 29.
- [9] W. Cassing et al., *Phys. Rep.* **188** (1990) 363.
- [10] J. Cugnon, T. Mizutani, and J. Vandermeulen, *Nucl. Phys.* **A352** (1981) 505.
- [11] J. Cugnon, D. Kinet, and J. Vandermeulen, *Nucl. Phys.* **A379** (1982) 553.
- [12] M. Cahay, J. Cugnon, and J. Vandermeulen, *Nucl. Phys.* **A411** (1983) 524.
- [13] J. Cugnon and M.-C. Lemaire, *Nucl. Phys.* **A489** (1988) 781.
- [14] H. Stöcker H. Sorge and W. Greiner, *Ann. Phys. (N.Y.)* **192** (1989) 266.
- [15] St. A. Baß, GSI report, GSI-93-13, 1993.

- [16] Gy. Wolf et al., *Nucl. Phys.* **A517** (1990) 615.
- [17] G. Wolf, W. Cassing, and U. Mosel, *Nucl. Phys.* **A545** (1992) 139c.
- [18] J.J. Molitoris and H. Stöcker, *Phys. Lett.* **B162** (1985) 47.
- [19] H. Stöcker and W. Greiner, *Phys. Rep.* **137** (1986) 287.
- [20] B.A. Li and W. Bauer, *Phys. Rev.* **C44** (1991) 450.
- [21] G.D. Harp et al., *Phys. Rev.* **C8** (1973) 581.
- [22] J.N. Ginocchio, *Phys. Rev.* **C17** (1978) 195.
- [23] Z. Fraenkel, E. Piasetzky, and G. Kalbermann, *Phys. Rev.* **C26** (1982) 1618.
- [24] D. Ashery and J.P. Schiffer, *Ann. Rev. Nucl. Part. Sci.* **36** (1986) 207.
- [25] H.J. Weyer, *Phys. Rev.* **195** (1990) 295.
- [26] C.H.Q. Ingram, *Nucl. Phys.* **A553** (1993) 573c.
- [27] L.L. Salcedo et al., *Nucl. Phys.* **A484** (1988) 557.
- [28] G.F. Bertsch and S. Das Gupta, *Phys. Rep.* **160** (1988) 189.
- [29] W. Cassing, K. Niita, and S.J. Wang, *Z. Phys.* **A331** (1988) 439.
- [30] S.J. Wang et al., *Ann. of Phys.* **209** (1991) 251.
- [31] W. Ehehalt et al., *Phys. Rev.* **C47** (1993) 2467.
- [32] Y. Horikawa, M. Thies, and F. Lenz, *Nucl. Phys.* **A345** (1980) 386.
- [33] W. Ehehalt et al., *Phys. Lett.* **B298** (1993) 31.
- [34] J.H. Koch, E.J. Moniz, and N. Ohtsuka, *Ann. of Phys.* **154** (1984) 99.
- [35] Partical Data Group, *Phys. Lett.* **B239** (1990) 1.

- [36] B.J. VerWest and R.A. Arndt, *Phys. Rev.* **C25** (1982) 1979.
- [37] M. Schäfer et al., Preprint UGI-93-5, Institute for Theoretical Physics, University of Giessen, 1993, to be published.
- [38] P. Danielewicz and G.F. Bertsch, *Nucl. Phys.* **A533** (1991) 712.
- [39] R.D. Ransome et al., *Phys. Rev.* **C45** (1992) R509.
- [40] K. Sneppen and C. Gaarde, Preprint NBI-93-14, The Niels Bohr Institute, University of Copenhagen, 1993.
- [41] A. Baldin et al., *Landold Börnstein* Vol. **12** , Springer, 1988.
- [42] V. Dimitriev, O. Sushkov, and C. Gaarde, *Nucl. Phys.* **A459** (1986) 503.
- [43] D. Ashery et al., *Phys. Rev.* **C23** (1981) 2173.
- [44] S.M. Levenson et al., *Phys. Rev.* **C28** (1983) 326.

Figure Captions

Fig. 1: Decomposition of the π^-p cross section: Breit–Wigner cross section for $\pi^-p \rightarrow \Delta^0$ (eq. (14)) (solid line) and Breit–Wigner cross section for $\pi^-p \rightarrow N(1440)$ (eq. (19)) (dashed line). Sum of eq. (14) and eq. (19) (dashed–dotted line). Experimental data are from ref. [41].

Fig. 2: Cross sections for the direct $pp \rightarrow pn\pi^+$ process based on the OBE model (see text) (solid line) and the direct $\pi^+(pn) \rightarrow pp$ process (25) (dashed line) as a function of \sqrt{s} .

Fig. 3: The mass dependent cross section for $pp \rightarrow \Delta^{++}n$ at $\sqrt{s} = 1.48$ in the OBE model from Schäfer et al. [37]. Experimental data are from ref. [42].

Fig. 4: The cross section for $\Delta^{++}n \rightarrow pp$ within different descriptions for the detailed balance as a function on $\sqrt{s} - M$: naive detailed balance eq. (30) (dashed line), calculation with the OBE model eq. (31) (dotted line), eq. (32) (solid line), Danielewicz [38] (dashed–dotted line). The calculations are for delta masses $M = 1180, 1232$ and $1280 MeV$.

Fig. 5: Mass dependent total absorption cross section for π^+ at $T_\pi = 165 MeV$ within different descriptions for the detailed balance: naive detailed balance of eq. (30) (dashed line), calculation with the OBE model eq. (31) (dotted line), eq. (32) (solid line), Danielewicz [38] (dashed–dotted line). Experimental data are from ref. [43].

Fig. 6: Total absorption cross section for π^+ on ^{12}C , ^{56}Fe and ^{209}Bi as a function of the pion energy: Calculation with the OBE model eq. (31) (dotted line), eq. (32) (solid line), Danielewicz [38] (dashed–dotted line). Experimental data are from ref. [43].

Fig. 7: Total absorption cross section for π^+ on ^{12}C and ^{209}Bi as function of the pion energy: Calculation with eq. (32) (solid line) and eq. (36) for the $\Delta N \rightarrow NN$ process (dashed line). Experimental data are from ref. [43].

Fig. 8: Mass dependent total inelastic cross section for π^+ at $T_\pi = 165\text{MeV}$: Calculation with the OBE model eq. (31) (dotted line), eq. (32) (solid line), Danielewicz [38] (dashed–dotted line). Experimental data are from [43].

Fig. 9: Energy dependence of the inelastic absorption cross section for π^+ on ^{12}C and ^{209}Bi : Calculation with the OBE model eq. (31) (dotted line), eq. (32) (solid line), Danielewicz [38] (dashed–dotted line). Experimental data are from ref. [43].

Fig. 10: Differential inelastic cross section for π^+ on ^{209}Bi (upper figure) and on ^{12}C (lower figure) at $T_\pi = 160\text{MeV}$. Calculation with isotropic delta decay (solid line) and with anisotropic decay eq. (21) (dashed line). Experimental data are from ref. [44].

Fig. 11: Total absorption cross section for π^+ on ^{12}C and ^{209}Bi as a function of the pion energy. Calculations with different delta rest masses are shown in the left figure and with different Γ_r in eq. (15) for all cross sections in the right figure. Experimental data are from ref. [43].

Fig. 12: Total absorption (upper figure) and total inelastic (lower figure) cross section for π^+ on ^{12}C as a function of the pion energy. Calculations with the parameters from ref. [40]: delta rest mass $M_\Delta = 1202\text{MeV}$ and $\Gamma_r = 200\text{MeV}$ only in $\Gamma(M)$ of eq. (20). Experimental data are from ref. [43].

Fig. 13: Energy dependence of the total absorption cross section for π^+ on ^{12}C and ^{209}Bi . Calculations with different delta potentials: $V_\Delta = V_N$ eq. (11) (solid line), shifted delta potential eq. (12) (dashed line) and potential from ref. [33] eq. (13)(dotted line). Experimental data are from ref. [43].

Fig. 14: Delta mass distribution for different delta generations for π^+ on ^{209}Bi at 85MeV (lower figure) and at 245MeV (upper figure). Free mass distribution eq. (29) (dashed line), first delta generation (solid line), second delta generation (dashed-dotted line), third delta generation (dotted line) and fourth delta generation (thick dashed line).